On the Possibilities of Predicting Cohort Fertility Measures from Period Fertility Measures: Theory and Empirical Evidence

P. C. Roger Cheng          Joshua R. Goldstein
What are period measures for?


- To estimate the fertility of cohorts
- Evidence is **mixed** on the validity of tempo-adjusted measures as estimators of completed cohort parameters (p851).
What are period measures for?


- This paper
  - establishes a formal relationship between period and cohort measures
  - responds to the literature casting doubts on the usefulness of period measures as cohort estimators
  - proposes three tempo-adjusted predictors of cohort quantum which are easy to implement
  - examines the performance in predicting the CTFR
the cohort-period relationship

$$CTFR(c) = \int_0^\beta f(a, c + a) da$$
the cohort-period relationship

PTFR(T) = \int_{0}^{\beta} f(a, T) \, da
the cohort-period relationship

\[ f(a, T) / PTFR(T) = p(a, T) \]

period fertility proportion

\[ PTFR(T) = \int_0^\beta f(a, T) da \]

\[ CTFR(c) = \int_0^\beta f(a, c + a) da \]

\[ = \int_0^\beta \frac{PTFR(c+a) \cdot p(a, c+a)}{1 - r(c+a)} \cdot da \]

a linear combination of PTFRs

\[ = \int_0^\beta BF(c+a) \cdot w(a, c+a) \cdot da \]

a linear combination of BFs
the cohort-period relationship

  - proposed an aggregate test of their formula
  - compared the completed CTFR with a weighted average of BF values over childbearing years
  - did not provide a formal inference and
  - the weights proposed differ from ours
\[ \text{CTFR}(c) = \int_0^\beta \text{BF}(c+a) \ w(a,c+a) \ da \]

✓ When data are available on completed childbearing, measuring cohort fertility is straightforward (Ni Bhrolchain, 2011, p.850).
CTFR\((c) = \int_{0}^{A} BF(c+a) \, w(a,c+a) \, da \quad \text{observed}
+ \int_{A}^{\beta} BF(c+a) \, w(a,c+a) \, da \quad \text{unfinished}

Propositions

\[ \int_{0}^{A} w(a,c+a) \, da + \int_{A}^{\beta} w(a,c+a) \, da = 1 \]

Freeze-BF1

\[ \int_{A}^{\beta} w(a,c+a) \, da = \int_{A}^{\beta} p(a,c+A) \, da \]

Freeze-BF2

\begin{align*}
\text{time} & \quad \text{age} \\
0 & \quad A & \quad \beta
\end{align*}
Define the truncation percentile as

\[
\alpha(A, c) = \frac{\int_0^A BF(c+a) \, w(a,c+a) \, da}{CTFR(c)}
\]

\[
CTFR(c) = \int_0^A f(a,c+a) \, da / \alpha(A, c)
\]

Observed

Unfinished

Proportion-Inflation
Empirical evaluation

- Compare the performance in predicting the CTFR
  - the Freeze-BF1
  - the Freeze-BF2
  - the Proportion-Inflation
  - the Freeze-Rate
  - the Linear-Extrapolation

- Data are from the HFD and the Eurostat
  - Canada, the U.S., and 23 European countries
  - 863 and 272 completed cohorts for all-birth-combined and parity-specific data
Empirical evaluation

- For each completed cohort, predict the CTFR based on partial information from age 15 through a chosen truncation age $A$ (varying between 19 and 43)
- Classify results by truncation percentile:
  - $[10,30)$
  - $[30,50)$
  - $[50,65)$ mean age of childbearing
  - $[65,75)$
  - $[75,85)$
Empirical evaluation

Adopt the prediction error index as:

\[
PE = \frac{\text{est. CTFR} - \text{true CTFR}}{\text{true CTFR} - \text{obs. CTFR}} \times 100\%
\]

- positive: overshoot
- negative: under-estimate

For example:

- true=2.0
- obs.=0.8
- est.=1.8

PE= - 16.67%

- true=2.0
- obs.=1.2
- est.=1.8

PE= - 25.00%

how much of the unfinished fertility has not been correctly estimated
Empirical evaluation

- Refering to Sobotka (2003, Table 12)

\[
\frac{\text{est. CTFR} - \text{true CTFR}}{\text{true CTFR}} \times 100\%
\]

<table>
<thead>
<tr>
<th></th>
<th>true CTFR</th>
<th>PE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very good</td>
<td>3%</td>
<td>7.5%</td>
</tr>
<tr>
<td>Good</td>
<td>5%</td>
<td>12.5%</td>
</tr>
<tr>
<td>Average</td>
<td>8%</td>
<td>20.0%</td>
</tr>
<tr>
<td>Poor</td>
<td>15%</td>
<td>37.5%</td>
</tr>
<tr>
<td>Very poor</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**PE** = \[
\frac{\text{est. CTFR} - \text{true CTFR}}{\text{true CTFR} - \text{obs. CTFR}} \times 100\%
\]
## Results

- **Average Performance of Absolute Prediction Error**

<table>
<thead>
<tr>
<th></th>
<th>very good</th>
<th>good</th>
<th>average</th>
<th>poor</th>
<th>very poor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.5%</td>
<td>12.5%</td>
<td>20.0%</td>
<td>37.5%</td>
<td></td>
</tr>
</tbody>
</table>

### Table 3: Average Performance by Method, Truncation Percentile, and Birth Order

<table>
<thead>
<tr>
<th>birth order</th>
<th>N</th>
<th>Freeze-BF1</th>
<th>Freeze-BF2</th>
<th>Proportion Inflation</th>
<th>Freeze-Rate</th>
<th>Linear Extrapolation</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
<td>2,627</td>
<td>13.57</td>
<td>13.46</td>
<td>13.32</td>
<td>17.40</td>
<td>18.66</td>
</tr>
<tr>
<td>1</td>
<td>585</td>
<td>6.17</td>
<td>6.04</td>
<td>4.99</td>
<td>10.19</td>
<td>11.74</td>
</tr>
<tr>
<td>2</td>
<td>700</td>
<td>9.31</td>
<td>9.15</td>
<td>7.82</td>
<td>11.91</td>
<td>15.94</td>
</tr>
<tr>
<td>3+</td>
<td>829</td>
<td>27.35</td>
<td>27.36</td>
<td>31.20</td>
<td>27.62</td>
<td>27.22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>truncation percentile ∈ [10%, 30%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3+</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>truncation percentile ∈ [30%, 50%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3+</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>truncation percentile ∈ [50%, 65%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3+</td>
</tr>
</tbody>
</table>

**TFR** 22.62 **BF**

- 28.75 20.32
- 24.34 13.07
- 29.43 18.65
- 29.04 21.15
- 29.88
Results

- Further examination across cohorts
  - select Canada, Netherlands, Sweden, and the U.S.
  - divide birth cohorts into three subgroups
    cohorts 1910-30, 1935-50, 1951-65
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  - select Canada, Netherlands, Sweden, and the U.S.
  - divide birth cohorts into three subgroups
    - cohorts 1910-30
    - cohorts 1935-50
    - cohorts 1951-65
Results: Box-whisker and [10%, 30%)

all-birth-combined

first birth

second birth

third birth and above
Conclusions

- The performance of CTFR estimators is mainly influenced by the quantum effect rather than the tempo effect.
- When the quantum effect is mild, our tempo-adjusted methods perform very well, particularly at a very young truncation age.
- As for cases when there exists a strong quantum effect, there may be no ideal method whose prediction of CTFR is statistically reliable.
Thanks for your time, comments welcome