Period Paramount or Cohort Key?
A cohort perspective on tempo adjustment

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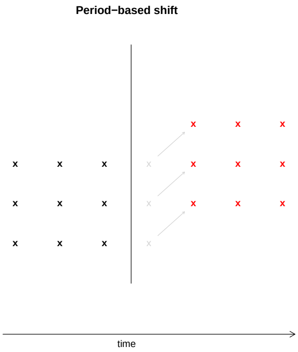
HFD Symposium

Agenda

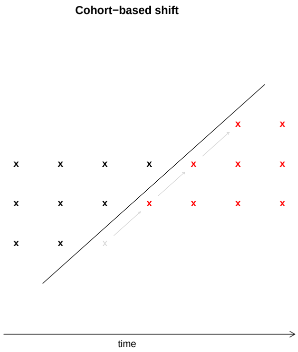
- Period vs. Cohort Shifts
- A “Movie”
- Formal Models
- Applications and Potential Advantages of Cohort Model

Tale of Two Tempo Shifts

Period Shift

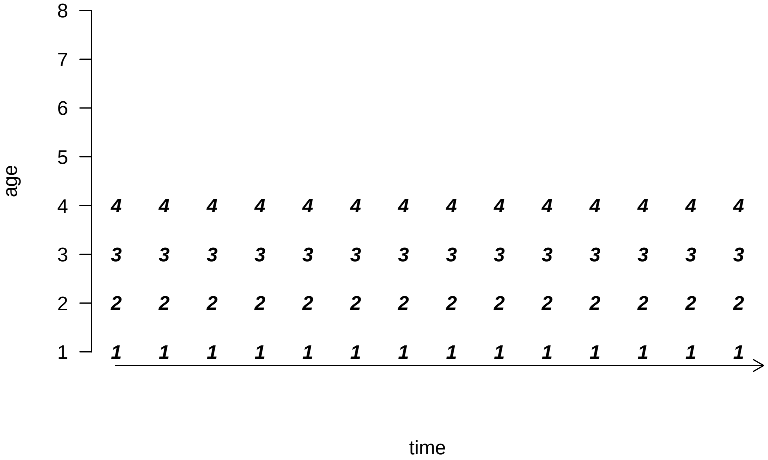


Cohort Shift



Our Cohort-shift model (Movie and Math)

A movie of cohort shifts



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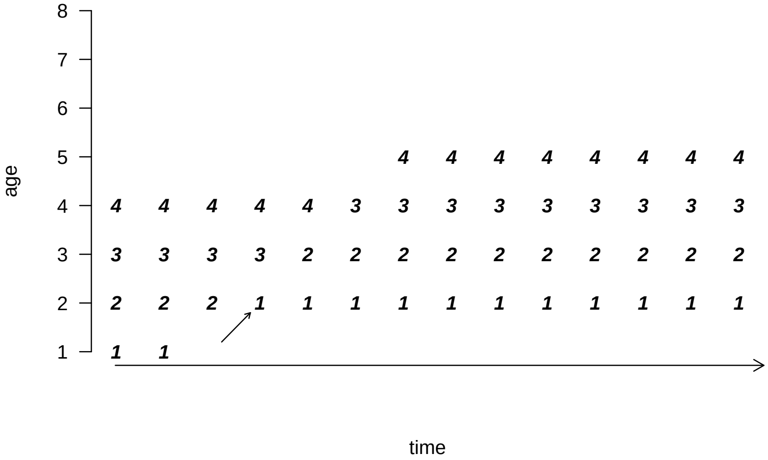

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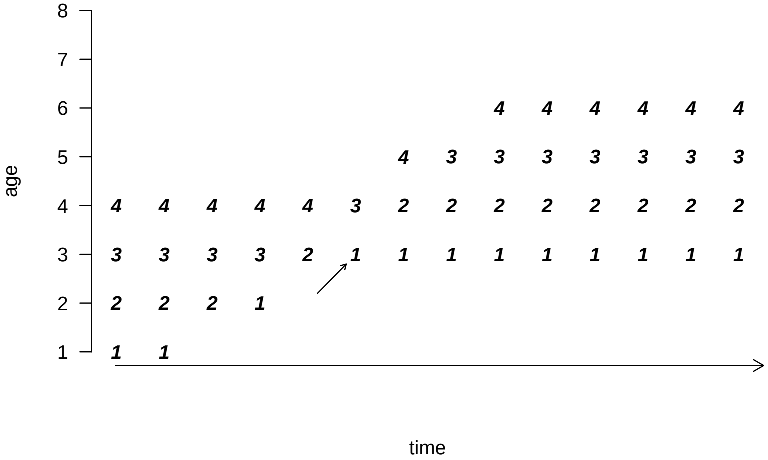

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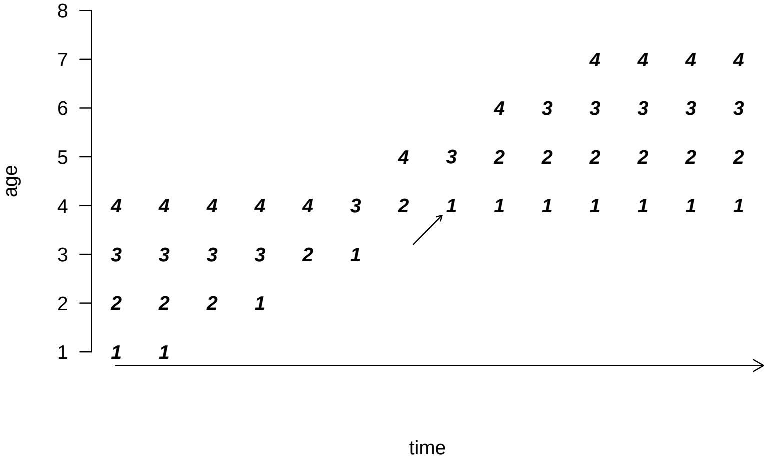

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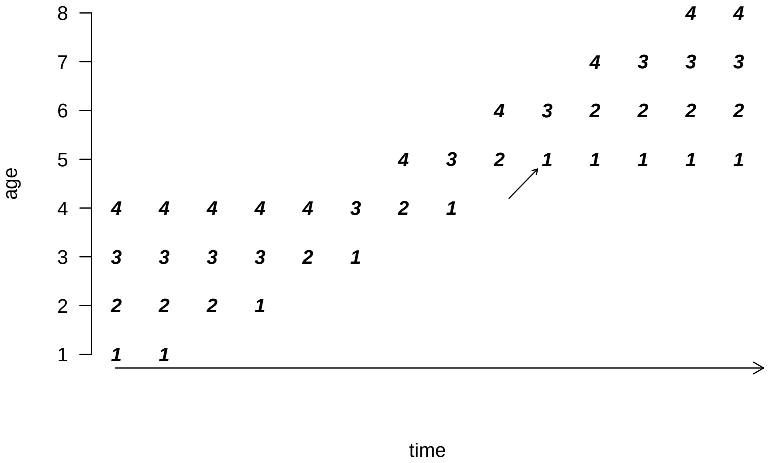

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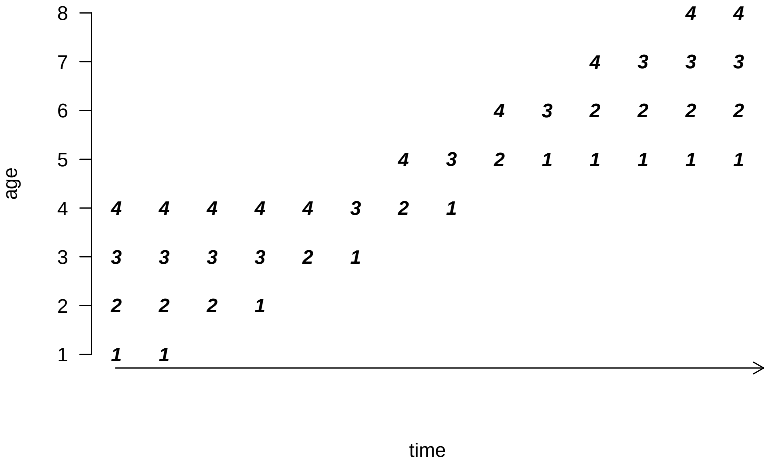

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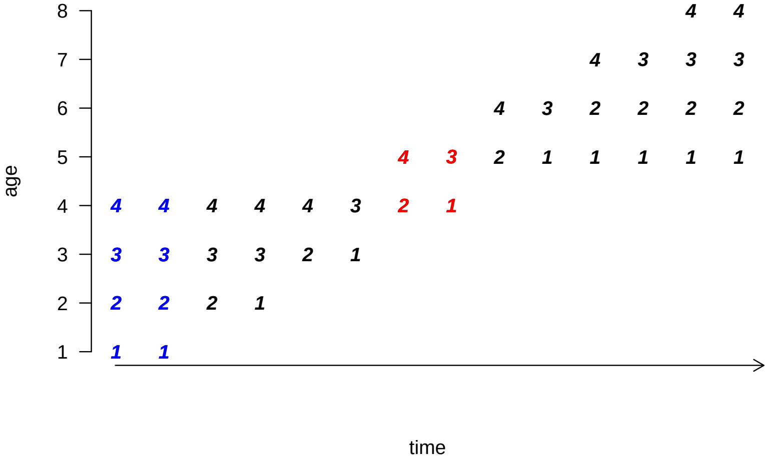

Formal description of shifts

Period shifts à la Bongaarts and Feeney.

$$f(a,t) = f_0(a - R(t)) \cdot (1 - R'_t)$$

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Our cohort shift model

$$f(a,t) = f_0(a - S(t - a))$$

Formal description of shifts *with period quantum*

Period shifts à la Bongaarts and Feeney

$$f(a,t) = \phi_0(a - R(t)) \cdot (1 - R'(t)) \cdot q(t)$$

(BF is period-based for both tempo and quantum)

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Period shifts à la Bongaarts and Feeney

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(BF is period-based for both tempo and quantum)

Our cohort-shift model

$$f(a,t) = \phi_0(a - S(t-a)) \cdot q(t)$$

(We are a hybrid: cohort tempo with period quantum)

Putting the cohort-shift model to use

1. Tempo-adjustment
2. Explain varying variance
3. Model fertility change statistically
4. See the future?

Tempo-adjusted (or "shift"-adjusted) period measures

Counterfactual: The measures give period quantum in the absence of shifts.

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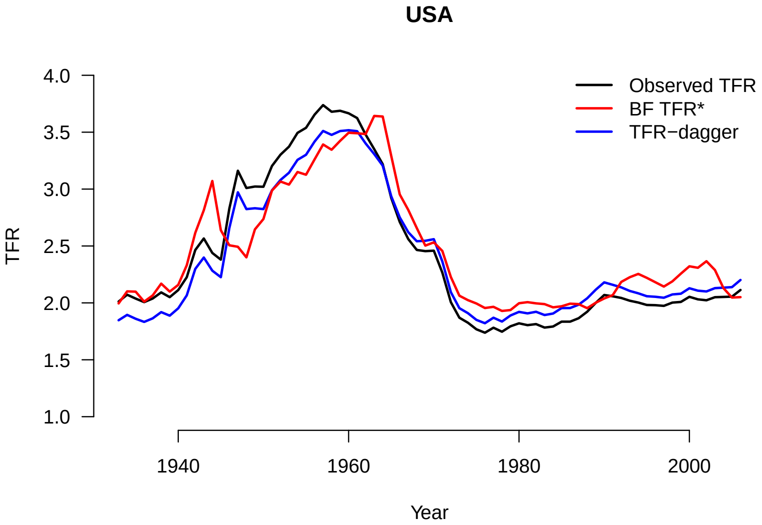
Adjustment un-does the age compressions introduced by cohort shifts.

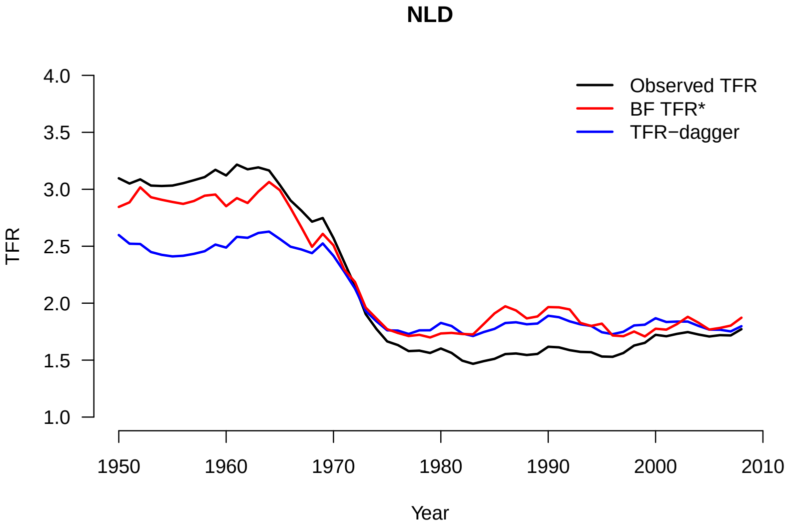
Four examples of recent fertility change



$TFR^{\dagger}$ gives less dramatic recent rise in fertility

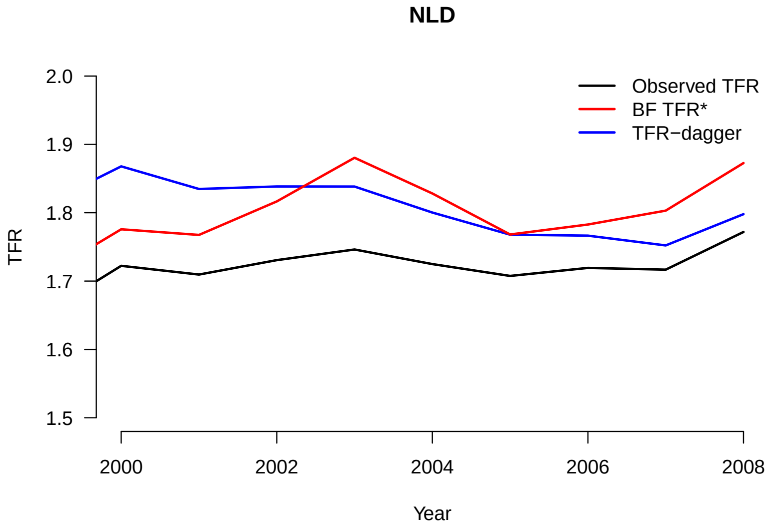
Does not rewrite story of babyboom

Four examples of recent fertility change



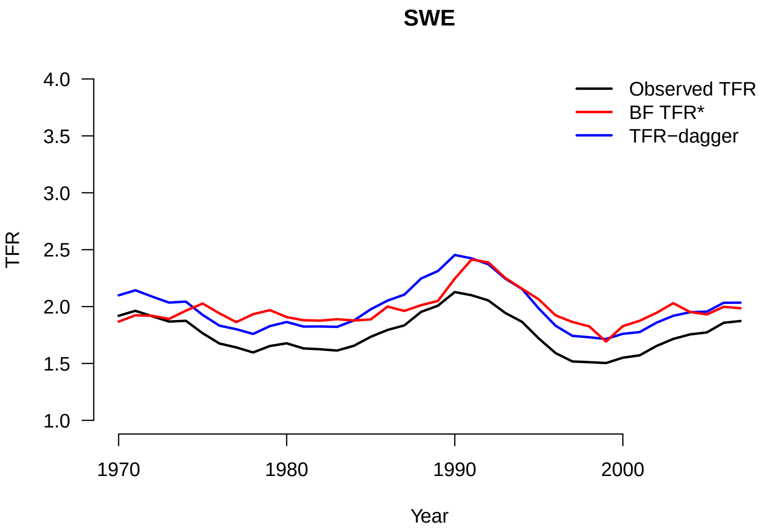
$TFR^{\dagger}$ has much smaller baby-boom and suggests no current tempo effect

Four examples of recent fertility change



$TFR^{\dagger}$ no current tempo effect

Four examples of recent fertility change



$TFR^\dagger$ echoes Kohler and Philipov's variance-adjusted TFR

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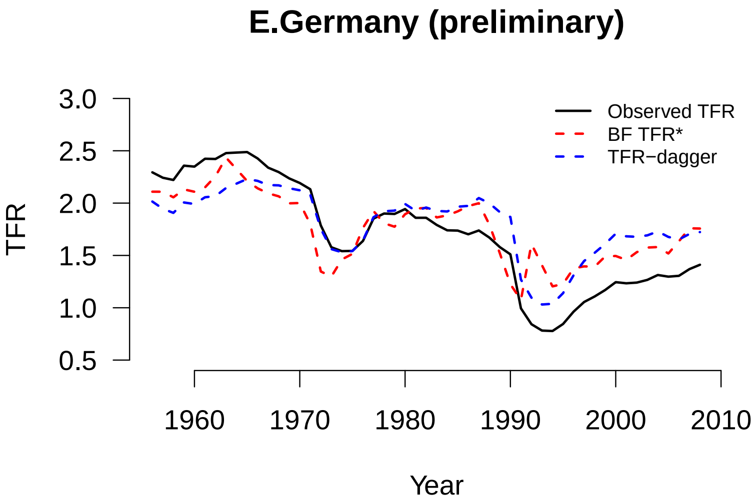

$TFR^{\dagger}$ suggests stability since c. 2000

Cohort shifts change period SD

Under linear cohort shifts, with constant quantum, period SD shrinks by same amount as TFR.

$$\mathrm{SD}_{per} = \frac{\mathrm{SD}_{coh}}{1 + S'}$$

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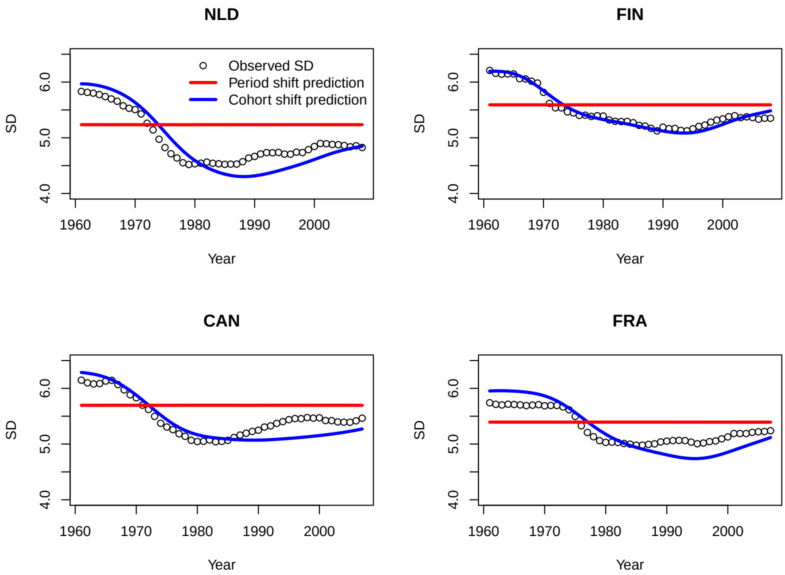
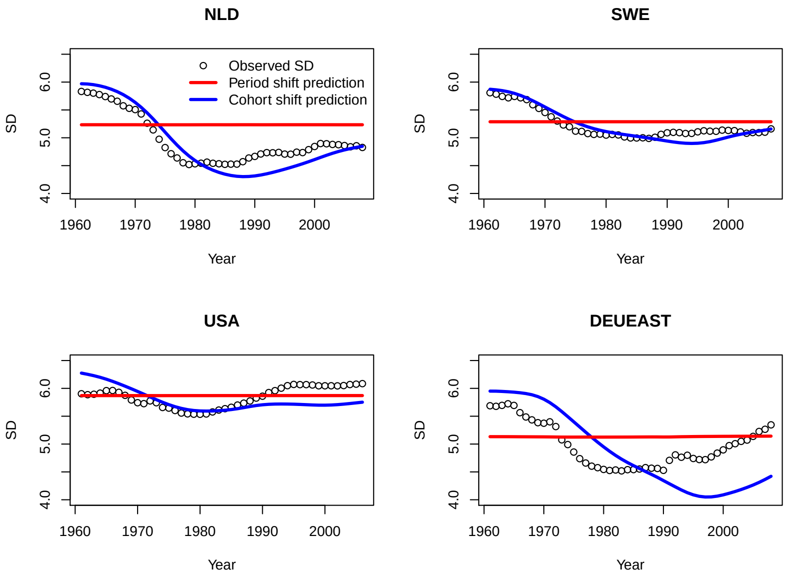
In general case, we can fit cohort-shift model numerically, and calculate implied SD.

Explaining varying variance



Explaining varying variance, more



Goodness-of-Fit Comparisons

Past validation was with cohort TFR, but a more direct test is to look at $f(a, t)$.
We have explicit formula for rates given parameters

$$\hat{f}^*(a, t) = \hat{f}_0^*(a - \hat{R}(t))(1 - \hat{R}'(t))\hat{q}^*(t)$$

$$\hat{f}^\dagger(a, t) = \hat{f}_0^\dagger(a - \hat{S}(t - a))\hat{q}^\dagger(t)$$

Can use computer optimization routines to find best f_0, given other parameters, or all of the parameters at once.
Then compare goodness-of-fit (e.g sum-of-squared residuals)

$$SSR = \sum_{a,t} \left(\hat{f}(a, t) - f_{obs}(a, t)\right)^2$$

Goodness-of-Fit Results

Table: Residual Sum of Squares from Semi-optimized Fitting, Netherlands

	1970-2008		
	period	cohort	$\frac{\text{cohort residiuals}}{\text{period residuals}}$
All parities, together $\sum_{a,t}\left(f-\hat{f}\right)^2$	0.082	**0.049**	61%
Parity 1	0.020	*0.010*	48%
Parity 2	0.010	*0.008*	82%
Parity 3+	0.004	*0.001*	37%

- Cohort model fits considerably better (smaller residuals)

“Postponement momentum”

Recall, under cohort shits,

$$f(a,t) = \phi_0(a - S(t-a)) \cdot q(t)$$

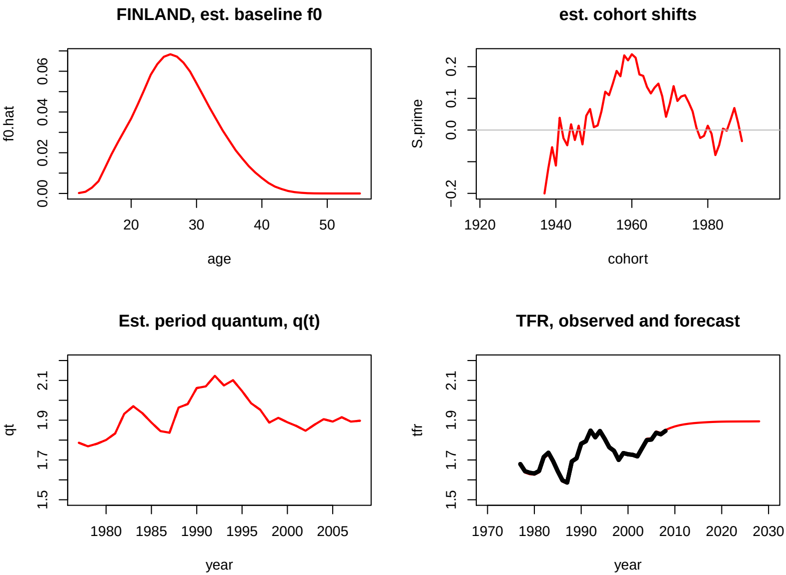
So,

$$TFR(t) = q(t) \int \phi_0(a - S(t-a))\, da$$

Tomorrow’s shifts will be very nearly today’s.

Contrast with period shifts, we must forecast both period level and entire period shift.

Momentum of cohort shifts, example



Conclusions

- Theory. Cohort shifts offer an alternative behavioral theory of tempo: shifting life course with period surprises.
- Math. Offers nice mathematical results with clear interpretation
- Model fitting. Optimization offers a new way to compare models, validate adjustments
- Some evidence that cohort model can be superior
  - Tells a clearer story (e.g., less recent boom)
  - Can fit better
  - Explains changing variance
  - Can aid forecasts

Future Research

- Combining period and cohort shifts
  (big deal for Eastern Europe)
- Using $TFR^{\dagger}$ as dependent variable for studying effects of economy, policy, etc.
- Studying postponement itself
- Considering other kinds of rates, by parity, hazards.

Thank you