ANALYZING THE LEVEL AND TIMING OF PERIOD FERTILITY

Robert Schoen

3145 Turk Blvd, #103
San Francisco CA 94118 USA
rschoen309@att.net

This paper was prepared for the November 3-4, 2011 Human Fertility Database Symposium on "Period and cohort fertility dynamics in the developed world," sponsored by the Max Planck Institute for Demographic Research, Rostock, Germany. Helpful comments from Vladimir Canudas-Romo are acknowledged with thanks.

Analyzing the Level and Timing of Period Fertility

ABSTRACT

Unprecedented changes in human fertility in recent decades have stimulated theoretical and methodological work, challenging the dominance of the cohort perspective and exploring the interaction of level and timing factors. Here, we first advance an approach to modeling timing effects using age-varying exponential weights, and provide in equation (5) a new relationship for incorporating timing effects into a given fertility schedule.

The implications of that approach highlight the inherent ambiguity in the terms "level" and "timing". In practice, those concepts can be neither uniquely specified nor disentangled. While both period and cohort perspectives enlighten fertility analyses, level and timing effects are substantively equivocal. Improvements in theory rather than in method are needed for a better understanding of the future course of fertility.

Analyzing the Level and Timing of Period Fertility

The emergence of sustained below replacement fertility in a number of developed countries has spurred research on the conceptualization and modeling of fertility behavior. The cohort perspective on fertility and its emphasis on completed family size, dominant since the days of Norman Ryder (1969, 1980, 1986), has been seriously challenged. Empirically, Hobcraft, Menken, and Preston (1982) found that period fluctuations have characterized most variations in fertility behavior. Conceptually, Ni Bhrolchain (1992) forcefully argued that period was paramount.

Associated with that re-examination of perspective, increased attention has been given to the level (or quantum) and timing (or tempo) components of fertility change. Contemporary period change in developed countries has frequently involved timing delays accompanied by lower fertility levels. Bongaarts and Feeney (1998) advanced an adjustment procedure that they claimed would eliminate "distortions" produced by timing effects, and argued that their adjusted measure implied a much more moderate decline in future cohort quantum than suggested by unadjusted period fertility measures. Others questioned the appropriateness of the Bongaarts-Feeney adjustment (Kohler and Ortega 2002; Kim and Schoen 1999, 2000) and/or the empirical implications drawn from using it (Frejka and Calot 2001; Lesthaeghe and Willems 1999; Van Imhoff and Keilman 2000). The controversy continues, as the issue of how to deal with period/cohort relationships and their timing and level aspects remains of substantive importance and scholarly interest (eg Ni Bhrolchain 2011).

The present paper has two objectives. The first is to set forth a method for modeling changes in period level and timing that can facilitate analyses of fertility change. The second is to reflect on the conceptual implications of that method, and to argue that the unavoidable conceptual ambiguity of the terms "level" and "timing" limit their value in understanding fertility dynamics.

MODELING CHANGES IN PERIOD LEVEL AND TIMING

The nature of timing changes in period fertility.

The literature reflects three principal ways to model changes in period fertility. The most common is to assume that fertility rates change proportionately at all ages. That is consistent with much empirical work (eg Hobcraft et al 1982), but leaves the period mean age at childbearing unchanged and hence is generally regarded as having no effect on fertility timing.

The second approach was used by Bongaarts and Feeney (1998), and essentially slides a fixed age pattern of fertility to higher (or lower) ages. For example, if there are postponements in period fertility of 0.2 years per year, then the fertility level at age x during time t applies to age x+0.2 at time t+1 and to age x+0.4 at time t+2. Over the 35 year reproductive period, that procedure would shift the fertility schedule upward by 7 years, an atypical pattern of change.

The third pattern is the one pursued here, and involves an exponentially increasing (or decreasing) factor applied to every age, with the rates at all ages then adjusted by a constant factor so that their sum yields the initial net reproduction rate (NRR). Specifically, let $\varphi_0(x)$ be the initial net maternity rate at exact age x, with net maternity scaled to replacement level so that the initial NRR = $\int \varphi_0(a)da = 1$. The timing adjusted net maternity rate at age x, $\varphi^*(x)$, is then given by

$$\varphi^*(x) = \varphi_0(x)\, e^{sx} \;/ \int \varphi_0(a)\, e^{sa}\, da \qquad (1)$$

where s is a parameter reflecting the extent of the timing adjustment. [Unless otherwise indicated, integrals range over all reproductive ages.] Because each age-specific numerator is divided by the sum of all numerators, the adjusted net reproduction rate (NRR*) is again 1, leaving the fertility level unchanged. If $s>0$, there is a delay, i.e. a relative shift in fertility to higher ages. The adjusted net maternity rates are then larger at high ages and smaller at low ages, as the adjustment procedure pivots the net maternity distribution around a mean age. If $s<0$, net maternity is accelerated as rates at the early ages increase.

The exponential adjustment of eq (1) is rooted in both observed behavior and stable population theory. Variations of that adjustment were used by Sykes (1973) and Kim and Schoen (1996) to analyze fertility change, and by Coale and Trussell (1974) in constructing their Model Fertility Schedules. The metastable population model, which generalizes the stable model to allow exponential change in fertility over age and time, uses an analogous transformation. Exponential net maternity change over age preserves the stable net maternity distribution, and is associated with a constant proportional distribution of births by age of mother (Schoen 2006, p121, 133).

<u>Implementing the exponential timing adjustment</u>.

The next step is to determine the mean age around which the net maternity distribution pivots. Under the Mean Value Theorem, there is some mean age, M, such that

$$NRR_{den} = \int \varphi_0(a)\, e^{sa}\, da = e^{sM} \int \varphi_0(a)\, da = e^{sM} \qquad (2)$$

as the φ_0 sum to 1.

Recall the stable population equality $R_0 = \exp(rT)$, where R_0 is the stable population net reproduction rate, r is Lotka's intrinsic growth rate, and T is Lotka's mean generation length (cf Schoen 2006, p10). Eq (2) indicates that M and s are analogous to Lotka's T and r. Moreover, in terms of the "crossover point" analysis of Kim and Schoen (1993), M is the point of intersection between net maternity densities φ_0 (stationary) and φ^* (stable). With s known, the value of M can be found can be found from the series

$$M = \mu - s\,\sigma^2/2 + s^2\,\mu_3/6 - \ldots \qquad (3)$$

where μ, σ^2, and μ_3 are the mean, variance, and third cumulant of the φ_0 distribution (cf. Keyfitz 1977, p126). Terms after the variance can generally be ignored. If one does not have s and M but only an overall factor, say F=exp[sM], then s can be found, using equation (3) through the variance term, from

$$s = [\ \mu - \{\ \mu^2 - 2\,\sigma^2 \ln F\}^{½}\ ] / \sigma^2 \qquad (4)$$

where ln indicates the natural logarithm. Equation (4) is analogous to Lotka's quadratic solution for intrinsic r (cf Schoen 2006, p10).

Hence, with net maternity pattern φ_0 and adjustment parameter s (or F), we can find the other parameters from eqs (3) and (4), and use eqs (1) and (2) to write the timing adjusted net maternity density φ^* as

$$\varphi^*(x) = \varphi_0(x)\, e^{s\,(x-M)} \qquad (5)$$

That exponentially adjusted net maternity distribution pivots around "generation length" M, which is determined by initial fertility pattern φ_0 and adjustment parameter s. With $s>0$, if $x<M$, then $\varphi^*(x) < \varphi_0(x)$, and when $x>M$, $\varphi^*(x) > \varphi_0(x)$. Equation (5) thus provides a straightforward and reasonable way to incorporate timing changes of any magnitude into fertility models.

Considering more general patterns of change in period fertility.

If net maternity does not change exponentially but by any arbitrary adjustment function g(x), then eq(1) generalizes to

$$\varphi^*(x) = \varphi_0(x)\, g(x) / \int \varphi_0(a)\, g(a)\, da \qquad (6)$$

Again applying the Mean Value Theorem

$$\text{NRR}_{\text{den}}\# = \int \varphi_0(a)\, g(a)\, da = g(A) \int \varphi_0(a)\, da = g(A) = e^{sB} = F \qquad (7)$$

where the last 2 equalities introduce parameters s, B, and F. When the adjustment to φ_0 was exponential, there was a unique mean age around which the net maternity distribution would pivot for a given F. In the more general case, however, pivoting age B need not be unique. Kim and Schoen (1993) found that multiple stationary-any crossover points could and often did exist in numerical applications. No general equation for determining those values is known, though they can be found from the φ_0 and φ^* distributions.

An illustrative calculation.

Table 1 shows the application of equation (5) to a fertility pattern based on that of United States Females, 1966. Adjustment parameter s is set at 0.005, and column (2) indicates that value implies a constant adjustment factor (e^{-sM}) of 0.8770. Column (5) shows the timing adjusted net maternity rates (φ^*). For age intervals under 25-29, net maternity falls, while for age intervals over 25-29, net maternity increases. Overall, the timing adjustment leaves the NRR unchanged.

Modeling changes in the level of period fertility.

Incorporating changes in fertility level has never been seen as problematic. It can readily be done by multiplying the fertility rates at every age by a constant factor.

Alternatively, the level of fertility can be changed by applying an exponential, age-weighted factor, e.g. e^{sx}, as was done in the previous section. Depending on whether s is greater or less than 0, fertility at all ages will increase or decrease, though by unequal proportions.

In short, either constant or age-weighted exponential factors can be used to reflect changes in the level of fertility over time. Both are demographically plausible.

Modeling simultaneous changes in level and timing.

As recent fertility behavior in developed countries has exhibited both delays and declines, an effective modeling approach must be able to accommodate both types of change simultaneously. The approaches described in the previous sections can do so.

Let us consider the implications of an exponential decline in fertility level of the form e^{-sx} (s>0) combined with an exponential delay that is also of size s. That doubly adjusted net maternity, φ^{**}, can then be expressed as

$$\varphi^{**}(x) = \varphi_0(x)\, e^{-sx}\, e^{s(x-M)} = \varphi_0(x)\, e^{-sM} \qquad (8)$$

Equation (8) shows that the combination of exponential delay and decline yields exactly the same adjusted net maternity as a decline that occurs proportionally at all ages. Thus an observed change in rates does not clearly specify the underlying level and timing dynamics. It follows that there is no unique or compelling way to decompose an observed fertility change into components reflecting level and timing influences.

In sum, equation (5) provides a new and useful approach for modeling specified timing changes, but represents an approach that facilitates analysis, not one that clarifies dynamics. Accordingly, we turn to questions of interpretation.

INTERPRETING CHANGES IN PERIOD FERTILITY

As a threshold matter, it is worth asserting and defending the proposition that both the period and cohort perspectives on fertility are valid and demographically significant. Cohort completed family size is theoretically meaningful, readily interpretable, and empirically stable. Period fertility reflects how cohort behavior unfolds. It determines birth cohort size, and as Schoen and Jonsson (2003) demonstrated, population size can decline even when every cohort more than replaces itself.

Given that reality synthesizes the period and cohort perspectives, what do the notions of fertility level and timing contribute? The argument here is that they are of limited value, and there are at least three reasons why that is the case. First, as Norman Ryder (1969) acknowledged over 40 years ago, quantum and tempo are structurally connected. Having more children takes more time. Second, the dual period and cohort perspectives confound the meanings of quantum and timing. A change in period timing produces a change in cohort quantum, and a change in cohort timing implies a change in period quantum. In short, any change simultaneously has both quantum and tempo effects.

Third, there is the message of equation (8). Even knowing the fertility rates at two points in time is not enough to uniquely determine the underlying patterns of quantum and tempo change, because the observed rates give only the outcome of the interaction between those two components. Multiple interpretations as to the nature of the underlying quantum and tempo changes are always possible, and there is no clear way to choose among them.

The inherent ambiguity in the terms level and timing is a serious concern only to the extent analyses focus on them in interpreting fertility change. Although an observed change cannot be definitively attributed to a timing rather than a level influence because multiple interpretations are always possible, there is no need to emphasize those components because demographers are fundamentally interested in the observed fertility behavior.

The fertility literature includes a number of measures that are clearly interpretable and that have stood the test of time, including essentially quantum measures like the TFR and NRR and essentially tempo measures like the mean age of fertility. Those summary measures are useful descriptive tools. While demographic measures can and should be improved, they have proven to be more than adequate for describing fertility behavior.

Predicting future behavior is another matter. That is not a methodological problem, but one that requires a theoretical answer. Elsewhere, colleagues and I have argued that the social resource or social capital value of children is the most important factor in determining fertility (Schoen, Kim, Nathanson, Fields, and Astone 1997; Astone, Nathanson, Schoen, and Kim 1999). At present, neither that view, nor any other proposed theoretical approach, can provide a firm basis for fertility projections. Still, we must look to improvements in theory for guidance in modeling future behavior, because improvements in methods and measures will not suffice.

Historically, one of the greatest spurs to theoretical development has been the collection and analysis of objective, reliable data. The efforts involved in creating and expanding the Human Fertility Database contribute to that end, and are producing a valuable resource whose exploitation can provide a better understanding of actual behavior and lead to new insights into the wellsprings of human fertility. I applaud the effort and expect that it will pay great dividends in the years ahead.

REFERENCES

Astone, N.M., Nathanson, C.A., Schoen, R., and Kim, Y.J. 1999. family demography, social theory, and investment in social capital. *Population and Development Review* 25: 1-31.

Bongaarts, J. and Feeney, G. 1998. On the quantum and tempo of fertility. *Population and Development Review* 24: 271-91.

Coale, A.J. and Trussell, T.J. 1974. Model fertility schedules: Variations in the age-structure of childbearing in human populations. *Population Index* 40: 185-258.

Frejka, T. and Calot, G. 2001. Cohort reproductive patterns in low-fertility countries. *Population and Development Review* 27: 103-32.

Hobcraft, J., Menken, J., and Preston, S. 1982. Age, period and cohort effects in demography: A review. *Population Index* 48: 4-43.

Keyfitz, N. 1977. *Introduction to the mathematics of population* (2d ed). Reading MA: Addison-Wesley.

Kim, Y.J. and Schoen, R. 1993. Crossovers that link populations with the same vital rates. M*athematical Population Studies* 4: 1-19.

Kim, Y.J. and Schoen, R. 1996. Populations with quadratic exponential growth. M*athematical Population Studies* 6: 19-33.

Kim, Y.J. and Schoen, R. 1999. Changes in timing and the measurement of fertility. Paper presented at the Annual Meeting of the Population Association of America in New York

Kim, Y.J. and Schoen, R. 2000. On the quantum and tempo of fertility: Limits to the Bongaarts-Feeney adjustment. *Population and Development Review* 26: 554-59.

Kohler, H.P. and Ortega, J.A. 2002. Tempo-adjusted period parity progression measures, fertility postponement, and completed cohort fertility. *Demographic Research* 6: 91-144

Lesthaeghe, R. and Willems, P. 1999. Is low fertility a temporary phenomenon in the European Union? *Population and Development Review* 25: 211-29.

Ni Bhrolchain, M. 1992. Period paramount? A critique of the cohort approach to fertility. *Population and Development Review* 18: 599-629.

Ni Bhrolchain, M. 2011. Tempo and the TFR. *Demography* 48: 841-61.

Ryder, N.B. 1969. The emergence of a modern fertility pattern: United States 1917-66. Pp. 99-126 in S.J. Behrman, L. Corsa, and R. Freedman (eds), *Fertility and family planning: A world view*. Ann Arbor: University of Michigan Press.

Ryder, N.B. 1980. Components of temporal variations in American fertility. Pp. 15-54 in R.W. Hiorns (ed), *Demographic patterns in developed societies*. London: Taylor and Francis.

Ryder, N.B. 1986. Observations on the history of cohort fertility in the United States. *Population and Development Review* 12: 617-43.

Schoen, R. 1988. *Modeling multigroup populations*. New York: Plenum.

Schoen, R. 2006. *Dynamic population models*. Dordrecht: Springer.

Schoen, R., Kim, Y.J, Nathanson, C.A., Fields, J. and Astone, N.M. 1997. Why do Americans want children? *Population and Development Review* 23: 333-58.

Schoen, R. and Jonsson, S.H. 2003. A diminishing population whose every cohort more than replaces itself. *Demographic Research* 9: 111-18.

Sykes, Z. 1973. Intrinsic age-specific birth rates: A new method of fertility analysis. Unpublished manuscript, Department of Population Dynamics, Johns Hopkins University, Baltimore.

Van Imhoff, E. and Keilman, N. 2000. On the quantum and tempo of fertility: Comment. *Population and Development Review* 26: 549-53.

Table 1. The Fertility Pattern of United States Females, 1966, Showing Adjustments for Changes in Level and Timing.

(1) Age	(2) Fertility Pattern (φ_0)	(3) Level Adjusted by a Factor of e^{-sM}	(4) Age Adjustment Factor $e^{-s(x+2.5)}$	(5) Timing Adjusted Fertility (φ^*)
10-14	.0016	.0014	.9394	.0015
15-19	.1299	.1139	.9162	.1243
20-24	.3411	.2991	.8936	.3348
25-29	.2731	.2395	.8715	.2748
30-34	.1561	.1319	.8500	.1610
35-39	.0761	.0667	.8290	.0805
40-44	.0208	.0183	.8086	.0226
45-49	.0013	.0011	.7886	.0014
TOTAL	1	.8770		1

NOTES. (1) Adjustment parameter s = 0.005, mean age of net maternity $\mu = 26.35$, variance of net maternity $\sigma^2 = 37.27$, and pseudo-generation length M = 26.26.

(2) The level adjustment factor is exp[−sM] = 0.8770. The value in column (3) is the entry in column (2) times 0.8770.

(3) The value in column (5) is 0.8770 times the entry in column (2) divided by the entry in column (4).

(4) All entries have been rounded independently. The calculation approximations underlying the entries in column (5) produce an overall error of 0.09%, which is ignored in the total shown.

SOURCE. Adapted from Schoen (1988, p44, Table 3.1).