Four mathematical models of fertility change

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Based our chapter in forthcoming Springer volume,
Dynamic Demographic Analysis (edited by Robert Schoen)

Kinds of explanation

- Cause and effect

  (identification revolution in economics)
- Understanding how people think

  (cognition, behavior, culture)
- Formal modeling

  (mathematics, often simple, to understand dynamics and properties of processes)

In Ron Lee’s words

Formal demography is nothing more than clear analytic thinking about a demographic problem, with hard-edged concepts, typically distilled into mathematical expressions.

Four models

1. Ryder's linear model

   (approximating polynomial change of age-specific fertility)

2. Lee's "moving target" model

   (changing fertility goals from period to period)

3. Bongaarts and Feeney's period-shift model

   (births being postponed from one period to the next)

4. Our own cohort-shift model

   (each cohort having its own shifted age-schedule)

Ryder's approach

Approximates a polynomial up to linear term

$$f(a,t) = f(a,0) + f'(a,0)\, t + \ldots$$

Result: period-cohort translation formula

Cohort TFR(c) ~ Period TFR($c + \mu_c$) / ($1 - \mu_c'$)

Lee’s approach

- Fertility target D(t) changes from year to year
- Fertility in each year is a flow of the difference between desired fertility in period and cumulative fertility to date
- Gives us

$$f(a,t) = \lambda [D(t) - C(a,t)]$$

- Results:
  - Tells how fertility will change targets change
  - Allows estimation of targets implicit in current rates

Bongaarts-Feeney, period shifts

Fertility the product of period quantum and period shifts in baseline fertility

$$f(a,t) = q(t)\, f_0(a - R(t))\,(1 - R'(t))$$

Note period quantum q(t), and period shifts R'(t)

Result: tempo-adjusted fertility

$$TFR^*(t) = TFR(t) / (1 - R'(t)) = TFR(t) / (1 - \mu'(t))$$

Goldstein-Cassidy, cohort shifts

Cohort shifts with period quantum

$$f(a,t) = q(t)\, f0(a - S(t-a))$$

Result: A tempo adjusted fertility that accounts for cohort shifts

$$\text{TFR-dagger}(t) = \int f(a,t)\, [\, 1 + S'(t-a)\,]\, da$$

Application to HFD

- Compare models (argue which is “best”)

- Allow each model to tell a different story

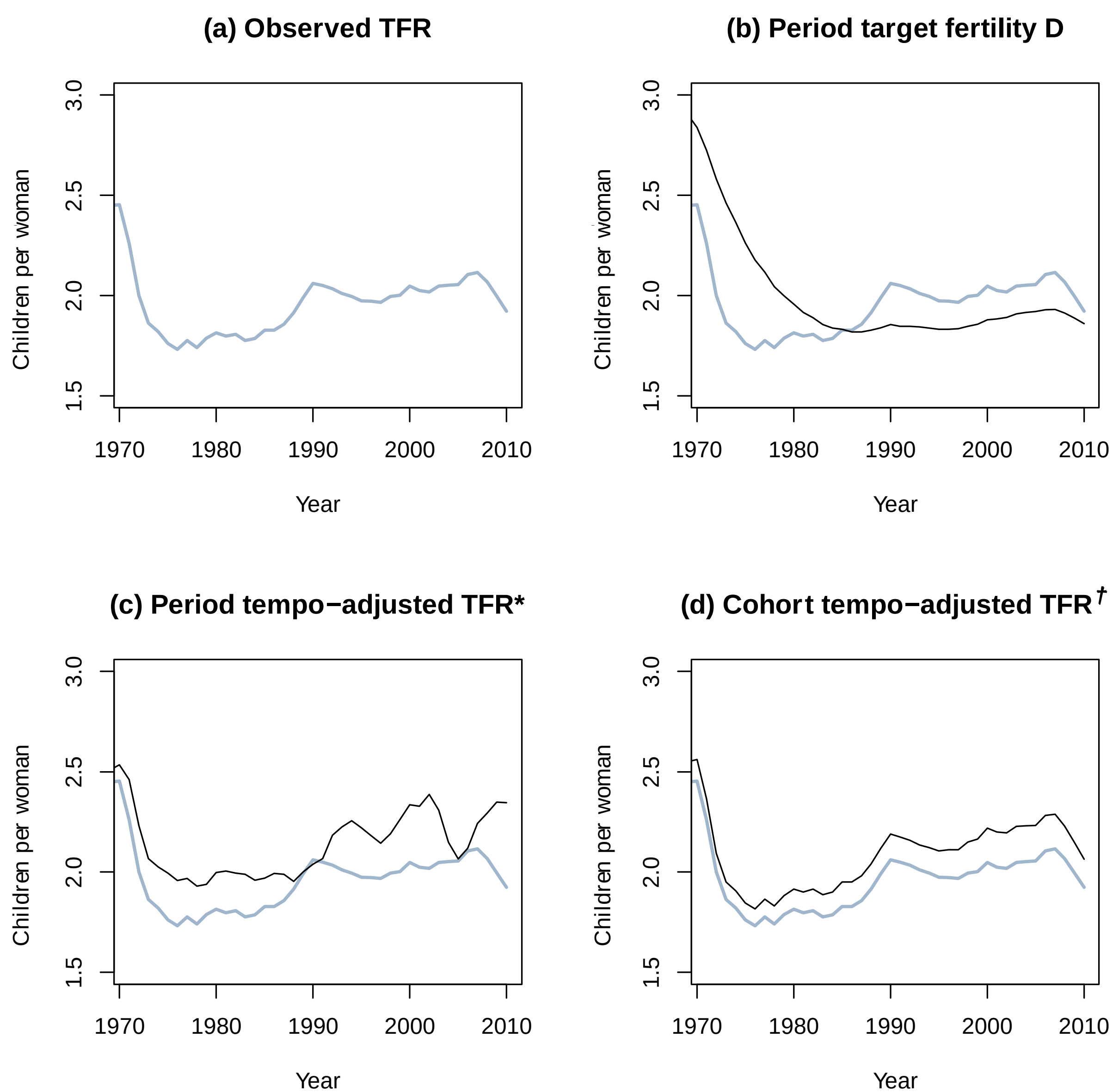
Using HFD data for goodness-of-fit

Table 1 Goodness-of-fit comparisons over two time intervals in Holland, with the smaller sum of squared errors (SSE) of fertility rates indicating a closer fit between the model and the observed data

	Holland 1960–2008		Holland 1975–2008	
	Period Fit SSE	Cohort Fit SSE	Period Fit SSE	Cohort Fit SSE
All Parities	0.231	**0.078**	0.039	**0.021**
Parity 1	**0.029**	0.038	0.013	**0.008**
Parity 2	**0.014**	0.018	0.007	**0.005**
Parity 3 +	0.009	**0.003**	0.001	0.001
All Parities, Fit Separately	0.097	**0.093**	0.038	**0.020**

Source: Goldstein and Cassidy (2014)

4 stories of U.S. fertility



Goldstein & Cassidy, forthcoming

Conclusions

- Value of model is not so much its fit
- But rather to give us a new way of thinking about fertility change
- Hope that HFD will be an important resource for creating new modeling approaches, new ways of gaining understanding