Period and cohort fertility dynamics in the developed world.
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On the Possibilities of Predicting Cohort Fertility Measures from Period Fertility Measures: Theory and Empirical Evidence

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What are period measures for?

Ni Bhrolchain, M (2011), Tempo and the TFR, *Demography* 48, p.841-861.

- To estimate the fertility of cohorts
- Evidence is **mixed** on the validity of tempo-adjusted measures as estimators of completed cohort parameters (p851).

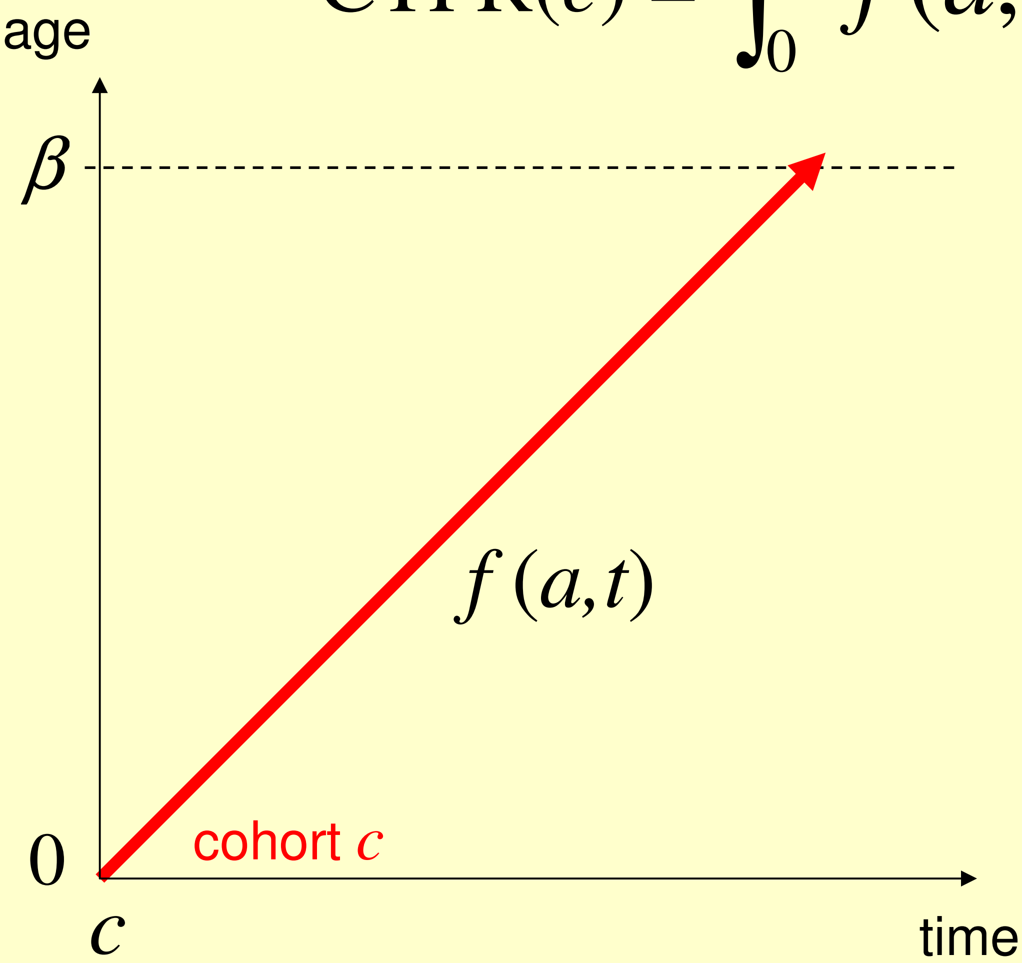
What are period measures for?

Ni Bhrolchain, M (2011), Tempo and the TFR, *Demography* 48, p.841-861.

- This paper
  - ✓ establishes a formal relationship between period and cohort measures
  - ✓ responds to the literature casting doubts on the usefulness of period measures as cohort estimators
  - ✓ proposes three tempo-adjusted predictors of cohort quantum which are easy to implement
  - ✓ examines the performance in predicting the CTFR

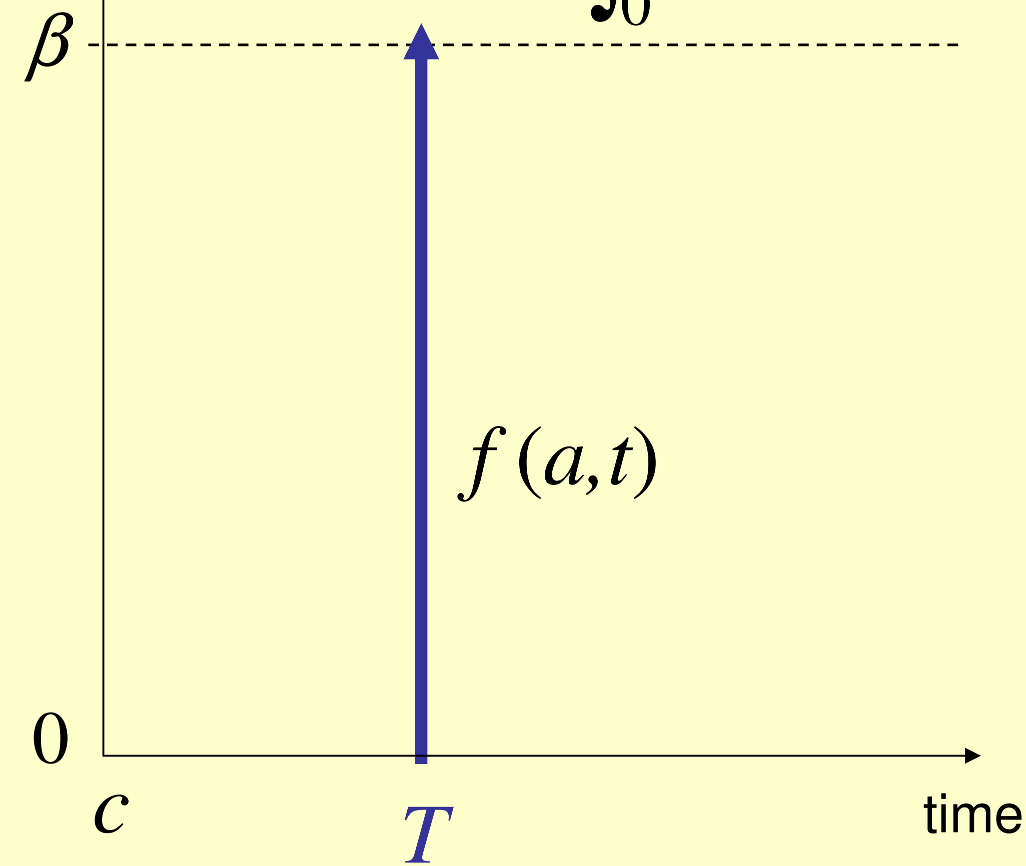
the cohort-period relationship

$$\text{CTFR}(c) = \int_0^{\beta} f(a, c+a)\,da$$



the cohort-period relationship

$$\mathrm{PTFR}(T) = \int_0^{\beta} f(a,T)\,da$$



the cohort-period relationship

$$f(a,T)\ /\ \mathrm{PTFR}(T) = p(a,T)$$

period fertility proportion

$$\mathrm{PTFR}(T) = \int_0^{\beta} f(a,T)\,da$$

$$f(a,t) = \mathrm{PTFR}(t)\ p(a,t)$$

$$\mathrm{CTFR}(c) = \int_0^{\beta} f(a, c+a)\,da$$

$$= \int_0^{\beta} \mathrm{PTFR}(c+a)\ p(a,c+a)\ da$$

a linear combination of PTFRs

$$= \int_0^{\beta} \boxed{\frac{\mathrm{PTFR}(c+a)}{1\text{-}r(c+a)}}\ \boxed{[1\text{-}r(c+a)]\ p(a,\ c+a)}\ da$$

$$= \int_0^{\beta} \mathrm{BF}(c+a)\ w(a,c+a)\ da$$

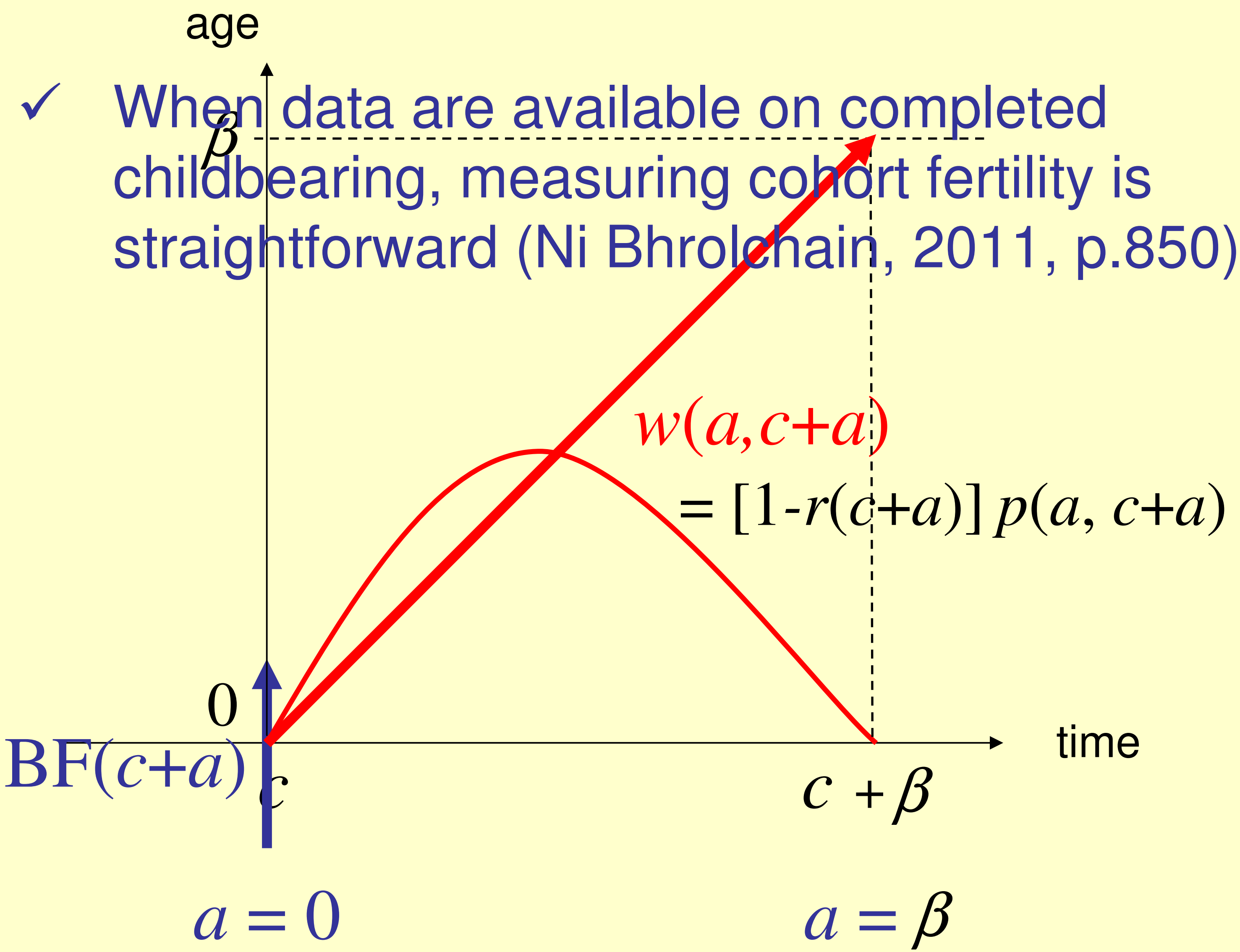
a linear combination of BFs

the cohort-period relationship

- Bongaarts and Feeney (1998, p.282-284)
  - ✓ proposed an aggregate test of their formula
  - ✓ compared the completed CTFR with a weighted average of BF values over childbearing years
  - ✓ did not provide a formal inference and
  - ✓ the weights proposed differ from ours

$$\mathrm{CTFR}(c) = \int_0^{\beta} \mathrm{BF}(c+a)\ w(a,c+a)\ da$$

- ✓ When data are available on completed childbearing, measuring cohort fertility is straightforward (Ni Bhrolchain, 2011, p.850).



$$\mathrm{CTFR}(c) = \int_0^A \mathrm{BF}(c+a)\, w(a,c+a)\, da$$ **observed**

$$+ \int_A^\beta \mathrm{BF}(c+A)\, w(a,c+a)\, da$$ **unfinished**

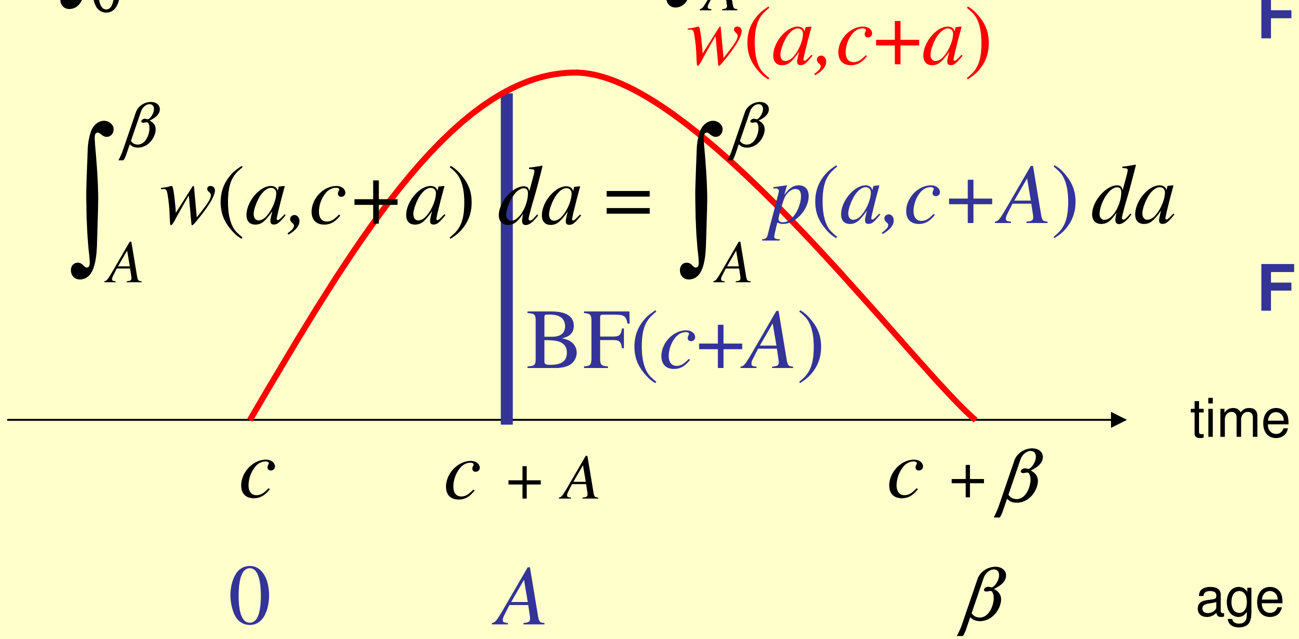
Propositions

$$\int_0^A w(a,c+a)\, da + \int_A^\beta w(a,c+a)\, da = 1$$

Freeze-BF1

$$\int_A^\beta w(a,c+a)\, da = \int_A^\beta p(a,c+A)\, da$$

Freeze-BF2



$$\mathrm{CTFR}(c) = \int_0^A \mathrm{BF}(c+a)\ w(a,c+a)\ da$$ **observed**

$$+ \int_A^\beta \mathrm{BF}(c+a)\ w(a,c+a)\ da$$ **unfinished**

Define the truncation percentile as

$$\alpha(A,\ c) = \int_0^A \mathrm{BF}(c+a)\ w(a,c+a)\ da \Big/ \mathrm{CTFR}(c)$$

$$\mathrm{CTFR}(c) = \int_0^A f\,(a,c+a)\, da \Big/ \boxed{\alpha(A,\ c)}$$

$$\int_0^A w\ (a,c+a)\, da$$

Proportion-Inflation

Empirical evaluation

- Compare the performance in predicting the CTFR
  - ✓ the Freeze-BF1
  - ✓ the Freeze-BF2
  - ✓ the Proportion-Inflation
  - ✓ the Freeze-Rate
  - ✓ the Linear-Extrapolation

- Data are from the HFD and the Eurostat
  - ✓ Canada, the U.S., and 23 European countries
  - ✓ 863 and 272 completed cohorts for all-birth-combined and parity-specific data

Empirical evaluation

- For each completed cohort, predict the CTFR based on partial information
- from age 15 through a chosen truncation age A (varying between 19 and 43)
- Classify results by truncation percentile:
  - [10,30)
  - [30,50)
  - [50,65) ← **mean age of childbearing**
  - [65,75)
  - [75,85)

Empirical evaluation

- Adopt the prediction error index as:

$$PE = \frac{\text{est. CTFR} - \textbf{true CTFR}}{\textbf{true CTFR} - \text{obs. CTFR}} * \mathbf{100\%}$$

- ✓ positive: overshoot
- ✓ negative: under-estimate
- ✓ For example:
  - true=2.0
  - obs.=0.8
  - est.=1.8

  PE= - 16.67%

- ✓ how much of the unfinished fertility has not been correctly estimated
  - true=2.0
  - obs.=1.2
  - est.=1.8

  PE= - 25.00%

Empirical evaluation

- Refering to Sobotka (2003, Table 12)

$$\frac{\left| \text{est. CTFR} - \textbf{true CTFR} \right|}{\textbf{true CTFR}} * 100\%$$

✓ very good	3%	→	7.5%
✓ good	5%		12.5%
✓ average	8%		20.0%
✓ poor	15%		37.5%
✓ very poor			

$$\text{PE} = \frac{\text{est. CTFR} - \textbf{true CTFR}}{\textbf{true CTFR} - \text{obs. CTFR}} * 100\%$$

Results

- **Average** Performance of **Absolute** Prediction Error

very good 7.5% good 12.5% average 20.0% poor 37.5% very poor

Table 3: Average Performance by Method, Truncation Percentile, and Birth Order

birth order	N	Freeze-BF1	Freeze-BF2	Proportion Inflation	Freeze-Rate	Linear Extrapolation	TFR	BF
				truncation percentile ∈ [10%, 30%)				
all	2,627	13.57	*13.46*	**13.32**	17.40	18.66		
1	585	6.17	*6.04*	**4.99**	10.19	11.74		
2	700	9.31	*9.15*	**7.82**	11.91	15.94		
3+	829	*27.35*	27.36	31.20	27.62	**27.22**		
				truncation percentile ∈ [30%, 50%)				
all	2,344	*13.98*	**13.71**	14.05	17.28	16.79		
1	507	*6.74*	*6.74*	**5.52**	11.74	11.03		
2	581	*10.28*	10.38	**8.69**	13.70	15.71		
3+	664	23.53	23.69	29.90	*23.38*	**22.62**		
				truncation percentile ∈ [50%, 65%)				
all	1,909	*14.35*	**14.05**	15.40	17.44	16.41	28.75	20.32
1	436	7.79	*7.65*	**6.49**	12.74	11.07	24.34	13.07
2	503	*10.83*	11.15	**9.61**	14.18	14.62	29.43	18.65
3+	530	22.09	22.20	31.48	*21.64*	**21.15**	29.04	29.88

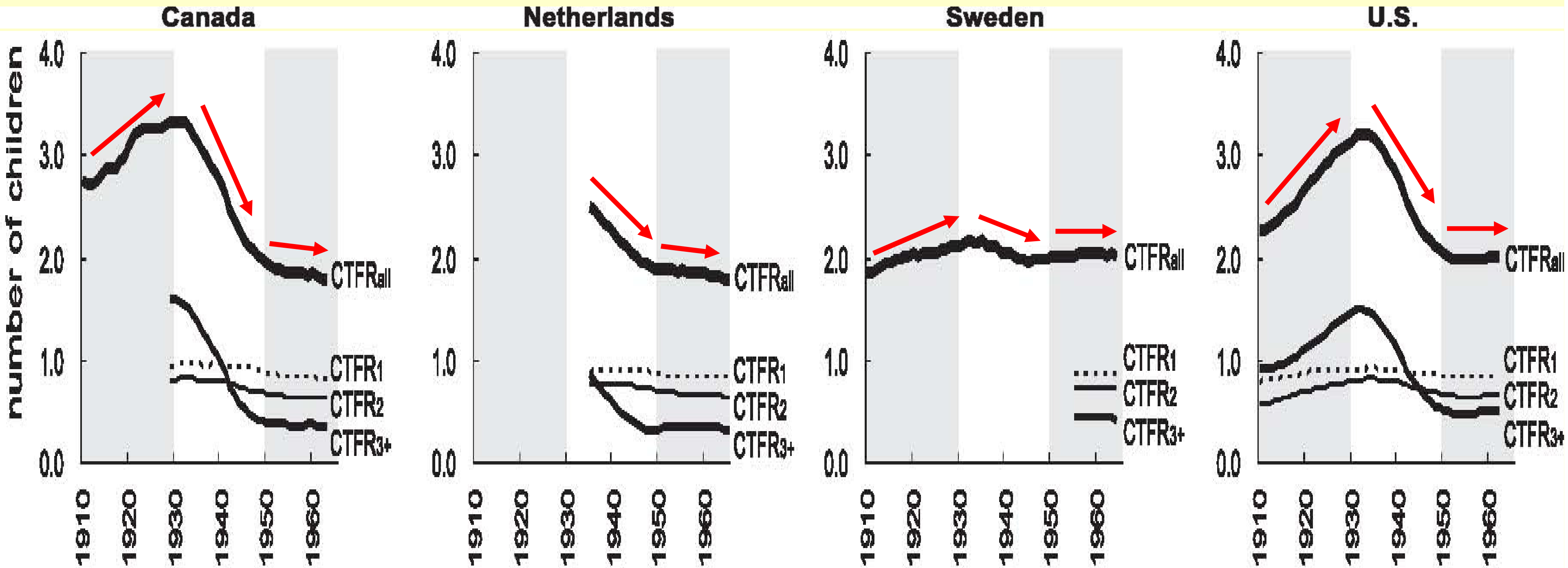
Results

➢ Further examination across cohorts

✓ select Canada, Netherlands, Sweden, and the U.S.

✓ divide birth cohorts into three subgroups

cohorts 1910-30, 1935-50, 1951-65



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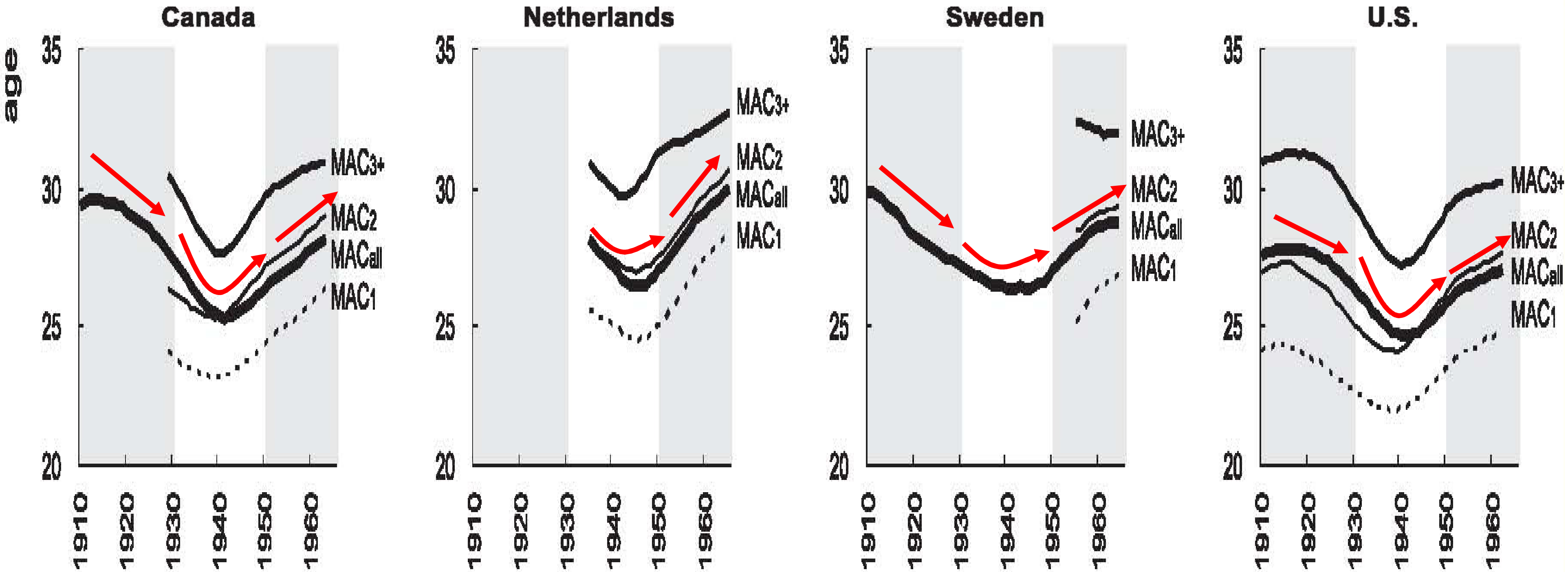
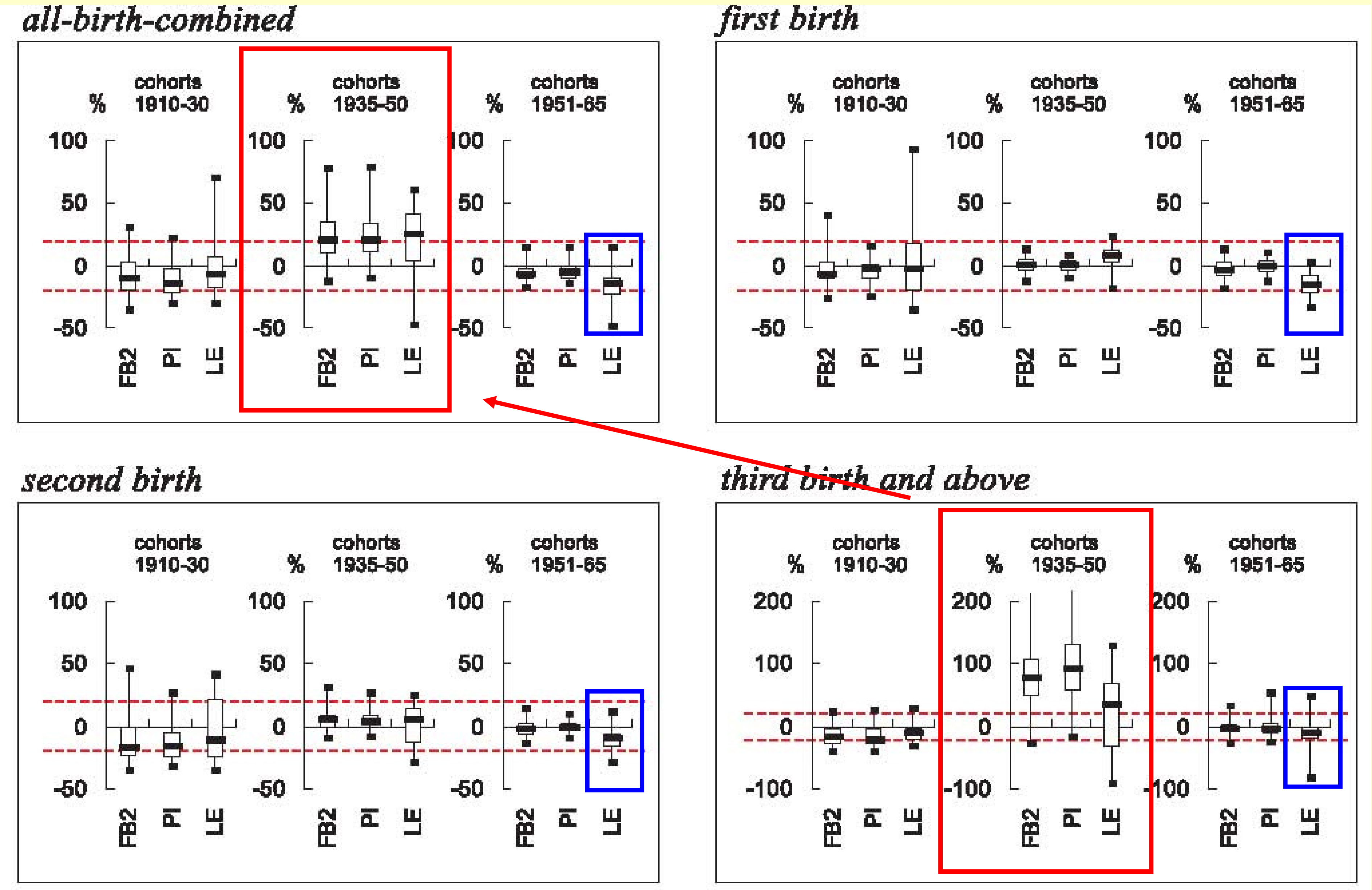


Results: Box-whisker and [10%,30%)



Conclusions

- The performance of CTFR estimators is mainly influenced by the **quantum** effect rather than the **tempo** effect.
- When the quantum effect is **mild**, our tempo-adjusted methods perform very well, particularly <u>at a very young truncation age</u>.
- As for cases when there exists a **strong** quantum effect, there may be **no** ideal method whose prediction of CTFR is **statistically reliable**.

Thanks for your time, comments welcome